For the limit
\[ \lim_{x \to 0} \frac{e^x - 1}{x} = 1, \]
illustrate the limit definition by finding a value of \( \delta \) that corresponds to \( \epsilon = .5 \).

\[ \left| \frac{e^x - 1}{x} - 1 \right| < .5 \]
or

\[ .5 < \frac{e^x - 1}{x} < 1.5 \]

Let's look at a graph of the function and the lower and upper bounds, .5 and 1.5, respectively.

\[ f := x \rightarrow \frac{e^x - 1}{x}; \]

\[ x \rightarrow \frac{e^x - 1}{x} \] (1)

\[ plot(\{f, .5, 1.5\}, -2 .. 2, -2 .. 2); \]
Note if $|x| < .5$, the function values are between the upper and lower bounds. Thus $.5$ is a possible value for $\delta$ when $\epsilon = .5$.

Let's try it for $\epsilon = .1$ In this case we have

\[ .9 < \frac{e^x - 1}{x} < 1.1 . \]

\[ plot( (f, .9, 1.1), -.5...5, .5..1.5) \]
A careful look at the graph shows that a value of \( \delta = .17 \) or less is appropriate for \( \epsilon = .1 \).

The following computations show the values of the function are within .1 of 1 for values of \( x \) within .17 of 0.

\[
\begin{align*}
|f(.1) - 1| &= 0.051709180 \\
|f(-.1) - 1| &= 0.0483741800 \\
|f(.15) - 1| &= 0.078894953 \\
|f(-.15) - 1| &= 0.0713865093
\end{align*}
\]
\[ |f(.17) - 1| = 0.090028535 \]
\[ |f(-.17) - 1| = 0.0803812741 \]