mat21ExamII

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. Differentiate the function.
   \[ g(x) = 4x^8 - 3x^5 + 7 \]
   \[ g'(x) = 32x^7 - 15x^4 \]
   a. \[ g'(x) = 32x^9 - 18x^5 + 7 \]
   b. \[ g'(x) = 32x^8 - 18x^6 \]
   c. \[ g'(x) = 32x^7 - 15x^4 \]
   d. \[ g'(x) = 28x^7 + 18x^6 \]

2. Differentiate.
   \[ f(x) = x^2 e^x \]
   a. \[ f'(x) = (x + 1)x e^x \]
   b. \[ f'(x) = (x + 2)x e^x \]
   c. \[ f'(x) = (x + 2)^2 x e^x \]
   d. \[ f'(x) = (x + 2)^2 e^x \]

3. Differentiate.
   \[ y = \frac{e^{4x}}{1 + 4x} \]
   a. \[ y' = \frac{e^{4x}}{1 + 4x} \]
   b. \[ y' = \frac{xe^{4x}}{(1 + 4x)^3} \]
   c. \[ y' = \frac{16xe^{4x}}{1 + 4x} \]
   d. \[ y' = \frac{16xe^{4x}}{(1 + 4x)^2} \]
4. Find the equation of the tangent line to the given curve at the specified point.

\[ y = 3xe^x, \ (0, 0) \]

To find \( m \)

\[ y - y_1 = m(x - x_1) \]

\[ y = 3xe^x + e^x \]

\[ y' = 3e^x (x+1) \]

\[ y' (0) = 3e^0 (0+1) = 3 \]

5. Differentiate.

\[ f(x) = x - 9 \sin x \]

a. \[ \frac{df(x)}{dx} = 1 - 9 \sin x \]

b. \[ \frac{df(x)}{dx} = 9 - \cos x \]

c. \[ \frac{df(x)}{dx} = 1 - 9 \cos x \]

d. \[ \frac{df(x)}{dx} = 9 - 3 \cos x \]

e. \[ \frac{df(x)}{dx} = 1 - \cos x \]

6. A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is \( x(t) = 3 \sin t \), where \( t \) is in seconds and \( x \) in centimeters. Find the velocity at time \( t \).

\[ x(t) = 3 \sin t \]

\[ v(t) = x'(t) = 3 \cos t \]

a. \( v(t) = \sin 3t \)

b. \( v(t) = 3 \sin 3t \)

c. \( v(t) = 3 \cos t \)

d. \( v(t) = \cos 3t \)
7. Suppose that \( F(x) = f(g(x)) \) and \( g'(14) = 11, \ g''(14) = 20, \ f'(14) = 13, \) and \( f'(11) = 5. \)

Find \( F'(14). \)

\[ F'(x) = f'(g(x)) \cdot g'(x) \]

\[ F'(14) = f'(g(14)) \cdot g'(14) \]

\[ F'(14) = f'(11) \cdot 20 = 5 \cdot 20 = 100 \]

8. Find \( y' \) by implicit differentiation:

\[ 6 \cos x \sin y = 12 \]

\[ 6 \cos(x) \sin(y(x)) = 12 \]

\[ \frac{d}{dx} \left[ 6 \cos(x) \cos(y(x)) \frac{dy}{dx} + (-\sin(x)) \sin(y(x)) \right] = 0 \]

\[ \frac{dy}{dx} = \frac{\sin(x) \sin(y(x))}{\cos(x) \cos(y(x))} \cdot \tan(x) \tan(y(x)) \]

9. Find an equation of the tangent line to the curve \( 3x^2 + 5y^2 = 8 \) at the point \((1, 1)\).

\[ y - y_1 = m(x - x_1) \]

\[ 6x + 10y \frac{dy}{dx} = 0 \]

\[ y - 1 = \frac{-3}{5} (x - 1) \]

\[ \frac{dy}{dx} = \frac{-6x}{10y} \]

\[ \left. \frac{dy}{dx} \right|_{(1, 1)} = \frac{-6}{10} = \frac{-3}{5} \]

\[ y - 1 = \frac{-3}{5}(x - 1) \]

\[ y = 6x + 1.6 \]

10. The equation of motion is given for a particle, where \( s \) is in meters and \( t \) is in seconds. Find the acceleration after 4 seconds.

\[ s = \sin 2 \pi t \]

\[ a = 0 \text{ m/s}^2 \]

\[ v = s' = 2 \pi \cos(2 \pi t) \]

\[ a = v'' = -2 \pi^2 \sin(2 \pi t) \]

\[ a(\pi) = -8 \pi^2 \sin(2 \pi \cdot \pi) \]

\[ a(\pi) = 0 \]
11. Differentiate the function:

\[ f(\theta) = \ln(\cos 5\theta) \]

(a) \[ f'(\theta) = -5 \tan(5\theta) \]

b. \[ f'(\theta) = -5 \cot(5\theta) \]

c. \[ f'(\theta) = \frac{5}{\cos(5\theta)} \]

d. \[ f'(\theta) = \frac{1}{\cos(5\theta)} \]

12. Use logarithmic differentiation to find the derivative of the function.

\[ y = x^{6x} \]

\[ y' = 6x^{6x}(6\ln x + 1) \]

b. \[ y' = 6(\ln x + 1) \]

c. \[ y' = 6x^{6x}(\ln x + 1) \]

13. Use the linear approximation of the function \( f(x) = \sqrt{6-x} \) at \( a = 0 \) to approximate the number \( \sqrt{5.99} \).

\[ \text{linear approximation} \]

\[ f(x) \approx f(0) + \frac{f'(0)}{2}(x-0) \]

\[ f(x) \approx \sqrt{6} - \frac{1}{2}(6-x)^{\frac{1}{2}} \]

Not \( \sqrt{5.99} \approx 2.447448565 \)

14. Find the differential of the function.

\[ y = x^9 + 4x \]

a. \[ dy = (9x^8 - 4)dx \]

b. \[ dy = (x^9 + 4)dx \]

c. \[ dy = (9x^8 + 4)dx \]

d. \[ dy = (9x^8 + 4)dx \]
15. Compute \( \Delta y \) and \( dy \) for the given values of \( x \) and \( dx = \Delta x \).

\[
y = x^2, \quad x = 1, \quad \Delta x = 0.5
\]

\[\Delta y = f(x+\Delta x) - f(x) = f(1.5) - f(1) = (1.5)^2 - 1 = 0.25\]

\[dy = f'(x)dx = 2x \cdot dx = 2 \cdot 0.5 = 1.0\]

(a) \( \Delta y = 1.25, \ dy = 1 \)
(b) \( \Delta y = 1.25, \ dy = 0 \)
(c) \( \Delta y = 0.25, \ dy = 1 \)
(d) \( \Delta y = 0.25, \ dy = 0 \)

16. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.09 cm thick to a hemispherical dome with diameter 55 m. (The surface area of a sphere is \(4\pi r^2\) where \(r\) is the radius.) Show all work. Use back if necessary.

\[V_{\text{hemi}} = \frac{2}{3} \pi r^3\]

\[\Delta V_H \approx \Delta V = 2 \pi (27.5)^2 \cdot 0.09 = 1.36125 \text{ m}^3\]

17. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure and volume satisfy the equation \(PV = C\), where \(C\) is a constant. Suppose that at a certain instant the volume is 550 cm\(^3\), the pressure is 155 kPa, and the pressure is increasing at a rate of 34 kPa/min. At what rate is the volume decreasing at this instant? Round the result to the nearest thousandth if necessary. (Note that both pressure and volume are functions of time. Show all work.)

\[P(t)V(t) = C\]

\[P'(t)V(t) + P(t)V'(t) = 0\]

\[V'(t) = -\frac{P'(t)V(t)}{P(t)}\]

18. If \(f(3) = 3, \ g(3) = 4, \ f'(3) = -5, \ g'(3) = 5\), find the following numbers.

\[(f+g)'(3) = \left(\frac{f(3)}{g(3)}\right)' = \frac{f'(3)g(3) + f(3)g'(3)}{g(3)^2} = \frac{-5 \cdot 4 + 3 \cdot 5}{4^2} = -2.1875\]

\[(f-g)'(3) = \frac{(f-g)(f-g)'}{(f-g)^2} = \frac{(-5)(-5) - 3(-10)}{1} = 35\]
19. Let \( f(x) \) denote a function that has at least six derivatives on an interval containing \( a \). Write the first five terms of the Taylor series of \( f(x) \) where the expansion is taken about the number \( a \).

\[
\begin{align*}
f(x) & \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \\
& \quad + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \frac{f^{(5)}(a)}{5!}(x-a)^5
\end{align*}
\]

20. Write a five term Taylor Series expansion for the function \( f(x) = \sin(x) \) where the expansion is taken about the number 0. (I.E., \( a = 0 \) in the expression in problem 19).

\[
\begin{array}{c|c}
f'(0) & \cos(0) = 1 \\
\hline
f''(0) & -\sin(0) = 0 \\
f'''(0) & -\cos(0) = -1 \\
f^{(4)}(0) & \sin(0) = 0 \\
f^{(5)}(0) & \cos(0) = 1
\end{array}
\]

\[
\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}
\]