(a) Show that of all rectangles with a given area \(A\), the one with the smallest perimeter \(P\) is a square.

Consider the rectangle with sides of length and width \(l\) and \(w\), respectively.

\[
A = lw \quad \text{must choose } l \text{ and } w \text{ such that } P = 2l + 2w \quad \text{is maximized and } A = lw \text{ holds.}
\]

Let's write \(P\) as a function of one variable \(l\). Using \(A = lw\), we have \(w = \frac{A}{l}\).

So

\[
P(l) = 2l + 2 \frac{A}{l}
\]

To maximize \(P\) we find \(l\) such that \(P'(l) = 0\)

\[
P'(l) = 2 - 2 \frac{A}{l^2}
\]

Since \(A = lw\), we have

\[
0 = 2 - 2 \frac{A}{l^2} \quad \Rightarrow \quad l^2 = A
\]

\[
l = \sqrt{A}
\]

Since \(l\) and \(w\) are the same, the figure is a square.