Find points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from $(1, 0)$.

Must maximize $(x-1)^2 + (y-0)^2$ where $(x, y)$ is on the ellipse.

Use ellipse equation to solve for $y$ in terms of $x$.

$$4x^2 + y^2 = 4$$

$$y = \sqrt{4 - 4x^2} = 2\sqrt{1-x^2}$$

The distance $(x-1)^2 + y^2$ can now be written as a function of $x$

$$d(x) = (x-1)^2 + 4 - 4x^2$$

or

$$d(x) = -3x^2 - 2x + 5$$

To find maximum, solve $d'(x) = 0$.

$$d'(x) = -6x - 2, \quad -6x - 2 = 0 \quad \text{so} \quad x = -\frac{1}{3}$$

The corresponding $y = 2\sqrt{1-x^2}$. So $y = 2\sqrt{\frac{2}{3}} = \frac{4\sqrt{2}}{3}$.

The points on the ellipse farthest from $(1, 0)$ are $\left(-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$.