1. Find all the critical numbers of the function:

   \[ g(x) = 6x + \sin(6x) \]
   
   \[ g'(x) = 6 + 6\cos(6x) \]

   a. \( \frac{\pi}{6} \)
   
   b. \( \frac{\pi(2n + 1)}{6} \)

   c. \( \frac{\pi(2n + 1)}{12} \)

2. Find the local minimum value of \( y = 5x^2 \) if \(-2 < x < 2\):

   \( y = 5x^2 \)
   
   \( y' = 10x \)
   
   \[ 10x = 0 \]
   
   \( \Rightarrow x = 0 \)

   \( (5, 0) \)

3. Find the critical numbers of the function:

   \[ y = 9x^2 + 36x \]

   a. \[ \frac{\pi}{6} \]
   
   b. \[ \frac{\pi(2n + 1)}{6} \]

4. Find the critical numbers of the function:

   \[ y = \frac{x}{x^2 + 25} \]

   a. \( 0, -5 \)
   
   b. \( 5, 0 \)

5. Find the absolute minimum values of:

   \[ y = 7x^2 - 56x + 2 \]

   \[ 14x - 56 = 0 \]
   
   \( 14x = 56 \)
   
   \( x = 4 \)

   (110)

   \[ x = 4 \]

   a. \( 0 \)
   
   b. \( 7.5 - 52.5 + 2 \)

   \( = 112 - 224 + 2 \)

   \( = -110 \)

   y(4) = 7.16 - 56.4 + 2

   y(5) = 7.5^2 - 52.5 + 2

   \( = 175 - 280 + 2 \)

   y(5) = 7.5^2 - 52.5 + 2

   \( = -103 \)
6 A cubic function is a polynomial of degree 3; that is, it has a form
\[ y = ax^3 + bx^2 + cx + d \quad a \neq 0 \]
\[ y' = 3ax^2 + 2bx + c \quad \text{at local extrema} \]
y'(x) = 0, y'(x) can have at most 2 roots. There are 2 (at most) local extrema.

What is the maximum quantity of local extreme values of cubic function?

7 Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [0, 14].

\[ \frac{f(14) - f(0)}{14} = \frac{f(c)}{14} \]

8 Find the exact values of the numbers c that satisfy the conclusion of The Mean Value Theorem for the function \( f(x) = x^3 - 2x \) for the interval \([-2, 2]\).

\[ a. c = \pm 2\sqrt{3} \]
\[ b. c = \frac{2\sqrt{3}}{3} \]
\[ c. c = \pm \frac{2\sqrt{3}}{3} \]

9 Verify that the function satisfies the hypotheses of The Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of The Mean Value Theorem.

\[ f(x) = 3x^2 + 4x + 5, [-7, 7] \]
\[ f'(x) = 6x + 4 \]
\[ b. c = 4 \]
\[ f(147) = 349 + 28 + 5 \]
\[ = 147 + 28 + 5 \]
\[ = 180 \]
\[ f(-7) = 3, 49 - 28 + 5 \]
\[ = 124 \]

10 Find the limit.
\[ \lim_{x \to -10} \frac{2x + 5}{x - 10} = -15 \]

\[ f(7) - f(-7) = f'(x) (7 - (-7)) \]
\[ 180 - 124 = (6x + 4) \cdot 14 \]
\[ 0 = 6x \]
\[ 56 = (6x + 4) \cdot 14 \]
\[ 9 = 6x + 4 \]
11. Find the limit.

\[ \lim_{{x \to 0}} \frac{e^x - 1}{\sin 3x} = \lim_{{x \to 0}} \frac{e^x}{3 \cos(3x)} = \frac{1}{3} \]

\[ \lim_{{x \to 0}} \frac{\ln 3x}{x} = \lim_{{x \to 0}} \frac{1}{3x} \cdot 3 = \lim_{{x \to 0}} \left( \frac{1}{x} \right) = \infty \]

\[ \lim_{{x \to \infty}} x^{10} e^{-x^2} = \lim_{{x \to \infty}} \frac{X^{10}}{e^{x^2}} = 0 \]

12. Find the limit.

\[ \min (x+y) \text{ Subject to } x,y = 144 \]

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14. Find two positive numbers whose product is 144 and whose sum is a minimum.

\[ \frac{x+144}{x} \]

15. Find the point on the line \( y = 4x + 8 \) that is closest to the origin.
16. Suppose the line $y = 4x - 3$ is tangent to the curve $y = f(x)$ when $x = 10$. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 10$, find the second approximation $x_2$.

$$x_2 = x_1 - \frac{f'(x_1)}{f(x_1)}$$

Given $f(x) = 4x - 3$, we have $f'(x) = 4$. Using $x_1 = 10$,

$$f(10) = 4(10) - 3 = 37$$

$$f'(10) = 4$$

$$x_2 = 10 - \frac{4}{37} = \frac{366}{37} \approx 10.0811$$

17. Use Newton's method with the specified initial approximation $x_1$ to find $x_3$, the third approximation to the root of the given equation. (Give your answer to four decimal places.)

$$x^4 - 12 = 0, \quad x_1 = 4$$

a. $x_3 = 2.3912$

b. $x_3 = 4.5412$

c. $x_3 = 3.0469$

d. $x_3 = 3.1079$

18. Use Newton's method to approximate the given number, correct to eight decimal places:

a. 1.50980365

b. 1.50980364

c. 1.50980363

d. 1.50980366

19. The following algorithm was used by the ancient Babylonians to compute $\sqrt{a}$:

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$$

(You can derive it by applying Newton's method to the equation $x^2 - a = 0$.)

Use this algorithm to compute $\sqrt{677}$, correct to 4 decimal places.

Let $x_1 = 26$, $26^2 = 676$.

$$x_2 = \frac{1}{2} \left( x_1 + \frac{677}{26} \right) = \frac{1}{2} \left( 26 + 26 \right) = 26$$

$$x_3 = \frac{1}{2} \left( x_2 + \frac{677}{26} \right) = \frac{1}{2} \left( 26 + 26 \right) = 26$$

$$\sqrt{677} \approx 26.01922366$$

One iteration suffices.
20 Find the most general antiderivative of the function:
\[ f(x) = 6x^2 - 12x + 6 \]
\[ a. F(x) = 10x^5 - 24x^4 + 6x + C \]
\[ b. F(x) = 6x^3 - 12x^2 + 6x + C \]
\[ c. F(x) = 2x^3 - 6x^2 + 6x + C \]

21 Find the most general antiderivative of the function:
\[ f(x) = 9\cos x - 3\sin x \]
\[ a. F(x) = 9\sin x + 3\cos x + C \]
\[ b. F(x) = -9\sin x + 3\cos x + C \]
\[ c. F(x) = 9\sin x - 3\cos x + C \]

22 Find \( f' \):
\[ f''(x) = 12x + 36x^2 \]
\[ f'(x) = 6x^2 + 12x^3 + C \]
\[ a. f(x) = 2x^3 + 3x^4 + Cx + D \]
\[ b. f(x) = 6x^3 + 6x^4 + Cx + D \]
\[ c. f(x) = 4x^3 + 12x^4 + Cx + D \]

23 A particle moves along a straight line with velocity function \( v(t) = 2\sin(t) - 10\cos(t) \) and its initial displacement is \( s(0) = 3 \). Find its position function.
\[ a. s(t) = 5 + 2\cos(t) - 10\sin(t) \]
\[ b. s(t) = 5 - 2\cos(t) - 10\sin(t) \]
\[ c. s(t) = 2 - 2\cos(t) + 10\sin(t) \]
\[ d. s(t) = 3 - 2\cos(t) - 10\sin(t) \]
\[ s(t) = -2\cos(t) - 10\sin(t) + C \]
\[ s(0) = 3 = -2\cos(0) - 10\sin(0) + C \]
\[ 3 = -2 + C \]
24. By reading values from the given graph of \( f \), use five rectangles to find a lower estimate for the area from \( x = 0 \) to \( x = 10 \) under the given graph of \( f \). Round your answer to the nearest tenth.

25. Estimate to the hundredth the area from 1 to 5 under the graph of \( f(x) = \frac{3}{x} \) using four approximating rectangles and right endpoints.
   - a. 3.85
   - b. 2.11
   - c. 5.41
   - d. 2.69
   - e. 4.51
   - f. 2.79

26. Use a computer algebra system to find the area from 0 to 3 under the curve \( y = x^2 \).
   - a. 7.85
   - b. 15.38
   - c. 14.19
   - d. 2.84
   - e. 9.00

Remember:
\[ \int_{a}^{b} f'(x) \, dx \text{ is the area under the curve defined by } f(x) \text{ over the interval } [a, b]. \]