MAT241FinalFall07

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. Find all the critical numbers of the function:
   \[ g(x) = 3x + \sin(3x) \]
   a. \( \frac{\pi(2n + 1)}{6} \)
   b. \( \frac{\pi(2n + 1)}{3} \)
   c. \( \frac{\pi}{3} \)
   d. \( \frac{2\pi n}{3} \)

2. Find the absolute minimum values of
   \[ y = 8x^2 - 64x + 3 \]
   on the interval \([0, 5]\).
   a. 64
   b. 4
   c. -128
   d. -125

3. Find all numbers \( c \) that satisfy the conclusion of The Mean Value Theorem.
   \[ f(x) = 3x^2 + 5x + 2, [-7, 7] \]
   a. \( c = 0 \)
   b. \( c = 5 \)
   c. \( c = 3 \)
   d. \( c = 2 \)

4. How many real roots does the equation \( x^5 + 2x + 1 = 0 \) have?
   a. exactly one real root
   b. no real root
   c. exactly two real root
   d. exactly three real root
5. Find the intervals on which the following function \( f \) is increasing:

\[ f(x) = x^3 - 108x + 8 \]

a. \((-\infty, 6)\)
b. \((-\infty, -6), (6, \infty)\)
c. \((-\infty, -18), (18, \infty)\)
d. \(-6, 6\)
e. \((-6, \infty)\)

6. How many points of inflection are on the graph of the function:

\[ f(x) = 16x^3 + 2x^2 - 9x - 15 \]

a. 4
b. 1
c. 3
d. 2

e. Not enough information provided

7. Find the limit.

\[ \lim_{x \to -9} \frac{x^2 + 7x - 18}{x^2 + 9} \]

a. 7
b. \(-\infty\)
c. \(\infty\)
d. 0
e. -11

8. Find two positive numbers whose product is 100 and whose sum is a minimum.

a. 2, 50
b. 4, 25
c. 10, 10

d. Not enough information provided

9. Suppose the line \( y = 25x - 6 \) is tangent to the curve \( y = f(x) \) when \( x = -2 \). If Newton's method is used to locate a root of the equation \( f(x) = 0 \) and the initial approximation is \( x_1 = -2 \), find the second approximation \( x_2 \).

a. \( x_2 = \frac{6}{25} \)
b. \( x_2 = \frac{-6}{25} \)
c. \( x_2 = \frac{8}{25} \)
10. Find the most general antiderivative of the function:

\[ f(x) = 9x^2 - 6x + 10. \]

a. \[ F(x) = 9x^3 - 6x^2 + 10x + C \]
b. \[ F(x) = 3x^3 - 2x^2 + 10x + C \]
c. \[ F(x) = 15x^5 - 12x^4 + 10x + C \]

11. Find the most general antiderivative of the function:

\[ f(x) = \frac{6}{x}, \quad x \neq 0. \]

a. \[ F(x) = -\frac{1}{x} + C \]
b. \[ F(x) = -\frac{1}{x^8} + C \]
c. \[ F(x) = \frac{1}{x^6} + C \]

12. Find the most general antiderivative of the function:

\[ f(x) = 3\cos x - 2\sin x \]

a. \[ F(x) = 3\sin(x) + 2\cos(x) + C \]
b. \[ F(x) = 3\sin(x) - 2\cos(x) + C \]
c. \[ F(x) = -3\sin(x) + 2\cos(x) + C \]

13. Find \( f \):

\[ f''(x) = 81 \cos(9x) \]

a. \[ f(x) = -\cos(9x) + Cx + D \]
b. \[ f(x) = y = 81\cos(x) + Cx + D \]
c. \[ f(x) = y = -\cos(9x) + Cx^2 + D \]
14. Find an expression for the area from 3 to 9 under the curve \( y = x^3 \) as a limit.

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( 3 + \frac{7i}{n} \right)^3 \frac{8}{n}
\]
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( 3 + \frac{8i}{n} \right)^3 \frac{5}{n}
\]
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( 3 + \frac{9i}{n} \right)^3 \frac{7}{n}
\]
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( 3 + \frac{6i}{n} \right)^3 \frac{6}{n}
\]
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( 3 + \frac{7i}{n} \right)^3 \frac{7}{n}
\]

15. Express the limit as a definite integral on the given interval.

\[
\lim_{x \to \infty} \sum_{i=1}^{n} \left[ 5x_i^2 - 15x_i \right] \Delta x , \quad [10,12]
\]

\[
\int_{10}^{12} \left( 5x^2 + 15x \right) \, dx
\]
\[
\int_{10}^{12} \left( 5x^2 - 15x \right) \, dx
\]
\[
\int_{5}^{15} \left( 10x^2 - 12x \right) \, dx
\]
16. If 1,800 cm$^2$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box. Enter your answer without units, and round to the nearest integer if necessary.

Answer __________________________

17. Use Newton’s method to derive the following algorithm used by the ancient Babylonians to compute $\sqrt{a}$:

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right).$$

(Hint: you can derive it by applying Newton's method to the equation $x^2 - a = 0$.)

18. A stone was dropped off a cliff and hit the ground with a speed of 256 ft/s. What is the height of the cliff?

Use a value of 32 ft/s for $g$, and enter the number of feet without the units.

Answer ______________________________
19. Estimate to the hundredth the area from 1 to 5 under the graph of \( f(x) = \frac{4}{x} \) using four approximating rectangles and right endpoints.

Answer

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**Short Answer**

20. Find the point on the line \( y = 2x + 9 \) that is closest to the origin.

Answer

21. Find \( f \):

\[ f''(x) = 60x \]

\[ f(1) = 15 \]

\[ f'(1) = 34 \]

Answer

22. A particle moves along a straight line with velocity function \( v(t) = 6 \sin(t) - 3 \cos(t) \) and its initial displacement is \( s(0) = 1 \) Find its position function.

Answer
MAT241FinalFall07
Answer Section

MULTIPLE CHOICE

1. ANS: B  PTS: 1
2. ANS: D  PTS: 1
3. ANS: A  PTS: 1
4. ANS: A  PTS: 1
5. ANS: B  PTS: 1
6. ANS: B  PTS: 1
7. ANS: E  PTS: 1
8. ANS: C  PTS: 1
9. ANS: A  PTS: 1
10. ANS: B  PTS: 1
11. ANS: A  PTS: 1
12. ANS: A  PTS: 1
13. ANS: A  PTS: 1
14. ANS: D  PTS: 1
15. ANS: B  PTS: 1

NUMERIC RESPONSE

16. ANS: 7,348  PTS: 1
17. ANS: 11.045361  PTS: 1
18. ANS: 1,024  PTS: 1
19. ANS: 5.13  PTS: 1

SHORT ANSWER

20. ANS: \( \left( -\frac{18}{5}, \frac{9}{5} \right) \)  PTS: 1
21. ANS:
   \[ f(x) = 10x^3 + 4x + 1 \]
   
   PTS: 1

22. ANS:
   \[ s(t) = 7 - 6\cos(t) - 3\sin(t) \]
   
   PTS: 1