Example 3 in section 5.1 shows the use of Riemann Sums to compute the area under the curve defined by $e^{-x}$ on $[0,2]$. In this example we use the Maple add function to compute the upper Riemann sums. We will evaluate the sums for partition sizes of 10, 100, 1000 and 10000. We will then compare the values obtained with the value of the definite integral.

If $n$ is the number of partitions, the upper Riemann sum on $[0,2]$ is given by

$$
\frac{2}{n} \sum_{i=1}^{n} e^{-\frac{2i}{n}}
$$

To evaluate this sum we define the function $f$.

$$
f := (n, i) \rightarrow e^{-\frac{2i}{n}};
$$

$$
\begin{align*}
(n, i) & \rightarrow e^{-\frac{2i}{n}} \\
(n, i) & \rightarrow e^{-\frac{2i}{n}}
\end{align*}
$$

The Maple add function will compute the sum. Note we must specify $n$.

In the following $n=10$. The add function sums the rectangles over $i$ where $i$ varies from 1 to 10. Here is the symbolic output of the add function.

$$
\text{add}(f(10, i), i = 1 .. 10);
$$

$$
e^{-\frac{1}{5}} + e^{-\frac{2}{5}} + e^{-\frac{3}{5}} + e^{-\frac{4}{5}} + e^{-1} + e^{-\frac{6}{5}} + e^{-\frac{7}{5}} + e^{-\frac{8}{5}} + e^{-\frac{9}{5}} + e^{-2}
$$

(3)

To calculate the sum we must multiply by $\frac{2}{n}$;

$$
\frac{2}{n}
$$

(4)

$$
\frac{2}{10} \cdot (\text{add}(f(10, i), i = 1 .. 10));
$$

$$
\frac{1}{5} e^{-\frac{1}{5}} + \frac{1}{5} e^{-\frac{2}{5}} + \frac{1}{5} e^{-\frac{3}{5}} + \frac{1}{5} e^{-\frac{4}{5}} + \frac{1}{5} e^{-1} + \frac{1}{5} e^{-\frac{6}{5}} + \frac{1}{5} e^{-\frac{7}{5}} + \frac{1}{5} e^{-\frac{8}{5}} + \frac{1}{5} e^{-\frac{9}{5}} + \frac{1}{5} e^{-2}
$$

(5)

To get numerical output, use the function evalf.

$$
\text{evalf}\left(\frac{2}{10} \cdot (\text{add}(f(10, i), i = 1 .. 10))\right);
$$

$$
0.7810785410
$$

(6)

Let's evaluate the sum for $n = 100, 1000, 10,000$ and 100,000.
\[ \text{evalf}\left(\frac{2}{100} \cdot \left(\text{add}(f(100, i), i = 1 \ldots 100)\right)\right) \]
\[ = 0.8560468913 \]  
\[ \text{(7)} \]

\[ \text{evalf}\left(\frac{2}{1000} \cdot \left(\text{add}(f(1000, i), i = 1 \ldots 1000)\right)\right) \]
\[ = 0.8638003396 \]  
\[ \text{(8)} \]

\[ \text{evalf}\left(\frac{2}{10000} \cdot \left(\text{add}(f(10000, i), i = 1 \ldots 10000)\right)\right) \]
\[ = 0.8645782584 \]  
\[ \text{(9)} \]

\[ \text{evalf}\left(\frac{2}{100000} \cdot \left(\text{add}(f(100000, i), i = 1 \ldots 100000)\right)\right) \]
\[ = 0.8646560755 \]  
\[ \text{(10)} \]

Now let’s compare these results with the definite integral.

Recall that the definite integral can be evaluated if we know the antiderivative. The antiderivative of \(e^{-x}\) is \(-e^{-x}\). Thus

\[ \int_{0}^{2} e^{-x} \, dx = -e^{-2} - (-e^{-0}); \]

\[ \text{evalf}\left(-e^{-2} - (-e^{-0})\right); \]
\[ = 0.8646647168 \]  
\[ \text{(11)} \]

Note that the approximation obtained using 100,000 rectangles in line is correct to 4 digits. (Compare lines (10) and (11)). The limit of the sums as \(n\) goes to infinity converges to the value in line (13).

Here is the direct calculation in Maple. Maple will evaluate the definite integral symbolically.

\[ \int_{0}^{2} e^{-x} \, dx \]
Use evalf to get a numerical approximation.

\[
\frac{e^2 - 1}{\ln(e) \cdot e^2}
\]

This is the same calculation we did using the definite integral. Maple just does not simplify the right side of the expression.