Feb. 7, 2008

p. 87, #3

\( P \left( 1, \frac{1}{2} \right) \) lies on the curve \( y = \frac{x}{1 + x} \).

(a) Find slopes of the secant lines for the following values of \( x \)
(i) .5, (ii) .9, (iii) .99, (iv) .9999

\[
\begin{align*}
\frac{x}{1 + x} & \quad (1) \\
\frac{f(1) - f(.5)}{1 - .5} & \quad 0.3333333334 \\
\frac{f(1) - f(.9)}{1 - .9} & \quad 0.263157895 \\
\frac{f(1) - f(.99)}{1 - .99} & \quad 0.25125628 \\
\frac{f(1) - f(.999)}{1 - .999} & \quad 0.2501251 \\
\end{align*}
\]

Let's also try .9999

\[
\frac{f(1) - f(.9999)}{1 - .9999} = 0.250013
\]

(b) It looks like the slope of the tangent is .25. (It is.)

(c) To get the tangent equation use the point slope form of the equation of a line where \( m = .25 \) and the point is \( \left( 1, \frac{1}{2} \right) \). The equation for the tangent is

\[
y - .5 = .25(x - 1)
\]
or

\[
y = .25x + .25
\]

Let's put the tangent into a maple function called \( t(x) \) and plot both the original curve and the tangent at \( x = 1 \).

\[
t := x \rightarrow .25x + .25;
\]
$y := t \rightarrow 40 \, t - 16 \, t^2$;

$y(2); \quad \frac{(y(2.5) - y(2))}{.5} = 16$  

$\frac{(y(2.1) - y(2))}{.1} = -25.6$  

$\frac{(y(2.05) - y(2))}{.05} = -24.800000$  

$\frac{(y(2.01) - y(2))}{.01} = -24.16$
\[
\frac{y(2.0001) - y(2))}{0.001} = -24.0016
\]  
\[\text{(13)}\]

The slope of the tangent at (2,16) appears to be -24. (It is.)
Using the point slope formula we can write the equation of the tangent.

\[y-16 = -24(t-2) \quad \text{or} \]
\[y = -24t + 64\]

Let's define a maple function for the tangent, denoted by \(g\), and plot the tangent and the original function.

\[g := t \rightarrow -24t + 64;\]
\[\text{plot} (y, g, -3..3);\]

\[\text{(14)}\]

p. 98, #19 Guess the limit as \(x \to 0\) of the following function.

\[f := x \rightarrow \frac{(e^x - 1 - x)}{x^2};\]
\( x \rightarrow \frac{e^x - 1 - x}{x^2} \) \hspace{1cm} (15)

\[ f(1); \]
\[ e - 2 \] \hspace{1cm} (16)

\[ f(0.1); \]
\[ 0.5170918000 \] \hspace{1cm} (17)

\[ f(0.01); \]
\[ 0.5016700000 \] \hspace{1cm} (18)

\[ f(0.001); \]
\[ 0.5000000000 \] \hspace{1cm} (19)

\[ plot(f, -1 .. 1); \]

\( f(0) \)

\textit{Error, (in f) numeric exception: division by zero}

Notice that the function is not defined at 0 but the limit as \( x \) approaches 0 is 0.5. Note also that the graph makes it look like the function is defined at 0. It is not. This is a quirk in Maple's graphing function. There should be an open circle around the point (0, 0.5).