Show \( \ln x = e^x \) has root on \((1, 2)\).  

Define \( F(x) = \ln(x) - e^x \).  

\[
F(1) = \ln(1) - e^1 = 0 - \frac{1}{e} < 0 \\
F(2) = \ln(2) - e^2 = \frac{1}{\ln(2)} - e^2 < 0 \\
F(3) = \ln(3) - e^3 = \frac{1}{\ln(3)} - e^3 > 0 \\
\]

Since \( F(1) < 0 \) and \( F(2) < 0 \) and \( F(3) > 0 \) there is by the Intermediate Value Theorem a root in \((2, 3)\).  

\[
F(e) = 0 \\
F\left(\frac{1}{2}\right) < 0 \\
\]